Volume 8, Issue 4

Fourth Quarter, 2003

The Quarterly Review Of Interest Rate Risk

Office of Supervision,
Economic Analysis Division

Special points of interest:

- 30-yr mortgage rate rises
- Yield curve shifts upward in the fourth quarter
- Fourth quarter median interest rate sensitivity increases
- Asset duration increases
- Comparative Regional Analysis
- Feature Article on Interest Rate Modeling and Financial Instrument Valuation

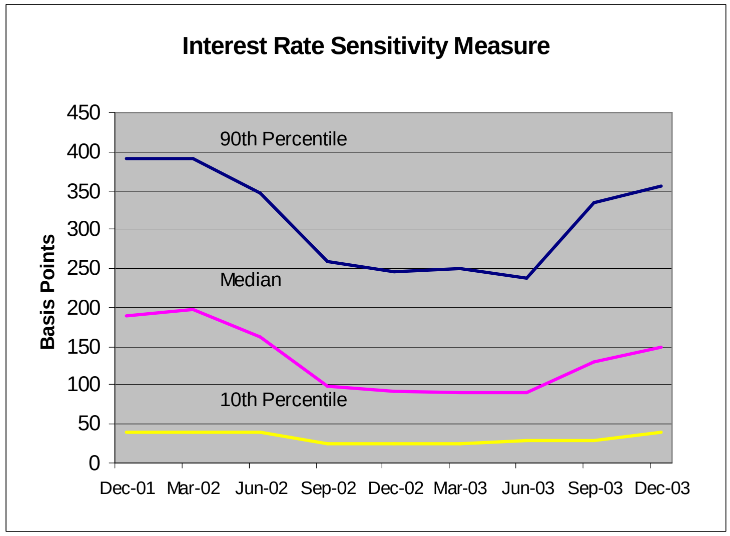
Charts in this issue:

Fourth Quarter Sees Interest Rate Sensitivity Rise

Median interest rate sensitivity increased from 133 basis points in the third quarter to 148 basis points in the fourth quarter. The rise in sensitivity for the thrift industry was due to the increase in interest rates in the fourth quarter.

Both the median pre–shock Net Portfolio Value (NPV) ratio and the median post-shock NPV ratios rose in the fourth quarter.

The number of thrifts with post-shock NPV ratios below 4 percent fell to five institutions, down from six in the previous quarter.



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Interest Rate Modeling and Financial Instrument Valuation

While all fixed-income financial instruments have values that vary with interest rates, many of these instruments, such as mortgage loans and mortgage-backed securities, have cash flows that are contingent on the path of future interest rates due to the prepayment options embedded in these instruments.

Interest rate financial derivatives, such as caps, floors, options on bonds, and swaptions, also have values that depend on the path of future interest rates. Only bonds that are both default- and option-free, such as a ten-year Treasury bond, have cash flows that are not interest rate path dependent, although their values do, of course, depend on the level of interest rates.

In order to calculate the present value of the expected cash flows associated with path-dependent interest rate sensitive instruments, it is necessary to have a model of some kind that offers a probabilistic description of the evolution of future interest rates.

This is because the value of a rate sensitive financial instrument must be calculated as the expected, or average, value of discounted cash flows over many alternative interest rate scenarios or paths.

Interest rate models offer the probabilistic, or stochastic, structure needed to produce future interest rate paths. By generating future interest rate paths, these models provide information on both the cash flows and the discount rates to be used in calculating the present discounted values of fixed-income instruments.

As such, interest rate models provide the key analytics of any fixed-income valuation and portfolio management system. In order to assess the return/risk tradeoff associated with fixed-income investments, it is important to properly derive prices or values for these securities.

For example, it is necessary to compute an option-

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Interest Rate Modeling and Financial Instrument Valuation (continued)

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adjusted spread (OAS) in order to value mortgages and mortgage-backed securities. OAS calculations require the use of an interest rate model for these path-dependent instruments.

Also, in order to effectively hedge a portfolio of fixed-income assets and liabilities, a portfolio manager must be able to value interest rate financial derivatives and derive the appropriate hedging weights (i.e., the deltas and gammas) associated with positions in these instruments. These hedging weights are important in portfolio rebalancing.

During the past 25 years, much research, both academic and proprietary, has focused on the development of interest rate models. Interest rate models have been developed by: Ho and Lee; Hull and White; Black, Derman, and Toy; Black and Karasinski; Heath, Jarrow and Morton; Cox, Ingersoll, and Ross; and Vasicek. These are just a few of the models currently used in valuing fixed-income securities and interest rate financial derivatives.

In order to understand fixed-income portfolio valuation, it is necessary to understand what an interest rate model is, what kinds of interest rate models are currently used in fixed-income and derivatives valuation, how different valuation methods are used to produce prices from the interest rate models, and how these models are used in portfolio management and interest rate risk measurement.

Classification of Interest Rate Models

For the most part, interest rate models are classified as either one– or two-factor models, depending on the number of stochastic factors that are used to model the dynamics of the term structure of interest rates. Typically, these models use interest rates as the stochastic factors, and they are modeled by stochastic differential equations, or "random walks."

In finance, stochastic differential equations are used to describe the movement in variables where part of the change in the value of the variable over time is purely random. These equations (also referred to as diffusion processes) are usually specified to consist of a non-stochastic drift term and a stochastic volatility term.

One-factor interest rate models have one stochastic factor, typically the short rate. In contrast, two-factor (or multifactor) interest rate models have two (or more) stochastic factors, typically the short rate and a long rate.

In addition to the stochastic or volatility components in these models, there is also a drift term that can be a deterministic function of time and which captures the mean change in the interest rate being modeled.

Typically, both the short and long interest rates in these models display mean-reversion, which means that when the rates are above (below) a long-term level, they should fall (rise) towards this level. Usually, the long-term level is a long-term average value.

By imposing an explicit mean-reverting drift to the short rate and long rate processes, it is possible to prevent excessive dispersion of the values that the interest rates can take on over time.

Perhaps somewhat surprisingly, it is possible to generate the entire term structure of interest rates from either one-factor or two-factor interest rate models. However, empirical work on the dynamics of the yield curve using principal components analysis has shown that one stochastic factor accounts for most of the change in the yield curve over time.

Principal components analysis is an estimation technique that can be used to determine the number of stochastic factors, or principal components, that explain most of the variation in a random variable.

A1991 study by Litterman and Scheinkman on common stochastic factors affecting U.S. Treasury bond returns (i.e., yield curve shifts) found that about 90 percent of yield curve movements are accounted for by one factor that explains level shifts. Typically, this one factor is modeled as the short interest rate. A second factor, typically modeled as the long interest rate, accounts for another 5 percent to 7 percent of the movement in the yield curve.

These results are consistent with the notion that the entire term structure of interest rates can be modeled with either a one- or two-factor model. The choice between one-factor and two-factor interest rate models depends on the type of fixed-income instrument that is to be valued.

For example, one-factor models are appropriate in valuing instruments that just depend on the short interest rate. On the other hand, both mortgage and mortgage-backed security valuations require a two-factor interest rate model, since prepayments are related to movements in a long term interest rate.

Once an interest rate model is chosen, there are several different methods that can be used to extract prices from the models in order to generate values of both path-dependent and non-path - dependent fixed-income financial instruments.

The three basic valuation approaches include Monte Carlo methods, finite difference approximations to the pricing equations associated with the stochastic differential equation representations, and lattice, or tree, methods.

Path-dependent securities, such as mortgage-backed securities, typically can only be valued using Monte Carlo techniques. Non-path-dependent securities, such as corporate callable bonds, can be valued using either finite differences or tree techniques. (See Rebonato, *Interest-*

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Interest Rate Modeling and Financial Instrument Valuation (continued)

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Rate Option Models, 1996,for a detailed discussion of these methods).

Besides the one-factor versus two-factor (multi-factor) distinction, interest rate models can also be categorized according to whether they are arbitrage-free or equilibrium models. This can be an important distinction, since it has implications for whether the model can be used for valuation and trading purposes. It is necessary to use arbitrage-free models for these purposes.

By definition, the arbitrage-free condition means that there exists no riskless strategy of zero net investment that yields a positive return with certainty. All arbitrage-free interest rate models are calibrated, or matched, to the current term structure of interest rates. This means that the bond prices produced, as output by an arbitrage-free interest rate model, are equal to the actual market prices of the bonds.

As a result, it is not possible to earn an arbitrage profit by trading bonds based on differences between market and model prices. Arbitrage-free interest rate models are described as being term-structure-consistent models, since they have parameters whose values exactly reproduce the current yield curve.

In contrast, equilibrium interest rate models produce term structures of interest rates that are only consistent with the equilibrium of an economy that is specified to have specific investor utility functions and production functions.

Equilibrium interest rate models are also arbitrage-free, but not in the same statistical sense as discussed above. Clearly, in order for an economic equilibrium to exist, there can be no arbitrage opportunities involving fixed-income or interest rate financial derivative instruments, but this does not mean that the interest rate model is calibrated, or matched, to the current yield curve.

Equilibrium interest rate models, however, can be converted to arbitrage-free interest rate models with a particular statistical adjustment, which allows the bond prices produced by the model to match the market prices of the bonds.

For example, the Cox, Ingersoll, and Ross interest rate model is an equilibrium model that can be converted to an arbitrage-free model with an adjustment that involves making the drift term in the stochastic differential equation time-varying.

Another distinction between interest rate models involves the distributional assumption imposed on the interest rate process. Some models specify the interest rate as being normally distributed, while other models specify a log-normal distribution for the interest rate.

This can be an important distinction. Normally distributed interest rates can assume negative values, while log-normally distributed interest rates can only assume positive values.

A simple example can be used to illustrate the importance of the distributional assumption invoked for interest rates for portfolio management.

It is possible to show, for example, that interest rate models with normally distributed interest rates tend to assign higher values to call options, thereby producing lower values for callable corporate bonds and most mortgage-backed securities.

On the other hand, interest rate models with log-normally distributed interest rates tend to value put options higher, thereby assigning higher values for corporate bonds with investor put options. (See Phoa, *Advanced Fixed Income Analytics*, 1998, for additional examples).

One-Factor Models

The general form of the stochastic differential equation that is used to model the short rate in a one-factor interest rate model is:

$$dr = u(r, t)dt + w(r, t)dX,$$

where r is the short rate, u(r, t) and w(r, t) characterize the behavior of the drift and volatility of the short rate, X is a Brownian motion stochastic process, and dr, dt, and dX denote small changes in r, t, and X.

By definition, the Brownian motion stochastic process, X, is assumed to be normally distributed with zero mean and variance dt. (See Karatzas and Shreve, *Brownian Motion and Stochastic Calculus*, 1991, for further discussion of Brownian motion).

In this general formulation, both the drift and volatility terms are allowed to be functions of both the level of the short rate and time. Different term structure shapes will be produced depending on the functional relationship specified for the drift and variance terms.

Some well-known examples of one-factor interest rate models include the arbitrage-free models of Ho and Lee; Hull and White; Black, Derman, and Toy; and Black and Karasinski; and the equilibrium models of Cox, Ingersoll, and Ross; and Vasicek.

There are several desirable properties that short-rate one-factor interest rate models should display. First, short rates should not be allowed to become negative with a high probability or to assume implausibly large values. Second, short rates should display mean-reverting behavior.

Third, the degree of correlation between rates of different maturity implied by the interest rate model should decrease more sharply at the short end of the yield curve than towards the long end.

Fourth, the volatilities of shorter interest rates should be higher than the volatilities of longer interest rates.

Finally, the short rate

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Interest Rate Modeling and Financial Instrument Valuation (continued)

(Continued from page 3)

volatility should vary positively with the level of the short rate.

Two-Factor Models

The general form of the stochastic differential equations that are used to model the short rate and the long rate in a two-factor interest rate model are:

dr = u(r, t)dt + w(r, t)dX

and

dl = v(l, t)dt + y(l, t)dZ,

where r and l are the short rate and the long rate, u(r, t) and w(r, t) characterize the behavior of the drift and volatility of the short rate, v(r, t) and y(r, t) characterize the behavior of the drift and volatility of the long rate, X and Z are Brownian motions, dt denotes a small change in t, dr and dX denote small changes in r and X, and dl and dZ denote small changes in l and Z.

As with the one-factor model, the two Brownian motions, X and Z, are assumed to be normally distributed with zero mean and variance dt.

The Vasicek Model

In order to better grasp the definitions and terminology above, it would be helpful to present an example of a simple interest rate model and how valuations are produced using it. The Vasicek model is a well-known equilibrium, one-factor interest rate model.

The specification of the short rate in this model is given by the following stochastic differential equation:

dr = b(c - r)dt + wdX

where r denotes the short rate, c denotes the long-term average value of the short rate, b is a positive parameter that denotes the mean reversion speed of the short rate around its long-run average value, w is the constant standard deviation of the short rate, and X is a Brownian motion.

Both the term structure of interest rates and the volatility structure for these rates are determined once the three parameters of the model, i.e., b, c, and w, are assigned values.

Suppose that a portfolio manager would like to value a European discount bond call option using the Vasicek model.

As it turns out, there are explicit pricing formulas for European discount bond call options that can be used to price these instruments. All that is required is to assign values to the three model parameters, and then values or prices for these instruments are generated.

Assume that the level of the short rate, r, is 5 percent, and that c=5 percent, b=15 percent, and w=1 percent per annum.

With these values for the Vasicek model, a European call option on a $100 five-year pure discount bond with a strike price of $67 would be $14.24. (See Clewlow and Strickland, *Implementing Derivatives Models,* 1999, for details). ¦

Fourth Quarter Sees Interest Rate Sensitivity Rise (continued)

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Treasury rates rose for all maturities in the fourth quarter, except for the three-month maturity. The increase for short-term and medium-term maturities between six months and five years was greater than for longer-term maturities.

In comparing the yield curve to that in the third quarter, it was more steeply sloped up to the three-year maturity point, but flatter for maturities greater than three years.

The Freddie Mac contract interest rate on commitments for fixed-rate 30-year mortgages increased to 5.85 percent at the end of the fourth quarter from 5.77 percent at the end of the previous quarter.

Although interest rates rose, thrift profitability was lower in the fourth quarter. The average return on assets for the industry fell to 1.20 percent from 1.28 percent in the prior quarter.

This decrease was attributed to lower non-interest income and higher non-interest expense in the fourth quarter.

The fourth quarter saw average net interest margin rise slightly to 285 basis points, up from 284 basis points in the third quarter. Thrift industry earnings rose to $3.45 billion in the fourth quarter, from $3.44 billion in the prior quarter.

In the fourth quarter, total fee income, which includes mortgage loan servicing fee income and other fee income, rose to 1.25 percent of average assets, up from 1.01 percent in the third quarter. Other fee income remained unchanged at 0.96 percent of average assets from the prior quarter. Other non-interest income fell to 0.46 percent of average assets from 0.76 percent between the third and fourth quarters.

The fourth quarter saw the ARM share of total thrift mortgage originations rise sharply to 37 percent, up from 26 percent in the prior quarter. Consistent with the rise in the share of thrift ARM originations, the ARM share of total 1-4 family mortgages held in portfolio rose to 60.6 percent from 55.4 percent in the prior quarter.

Fourth-quarter 1-4 family mortgage originations by thrifts fell dramatically to $143.9 bil-

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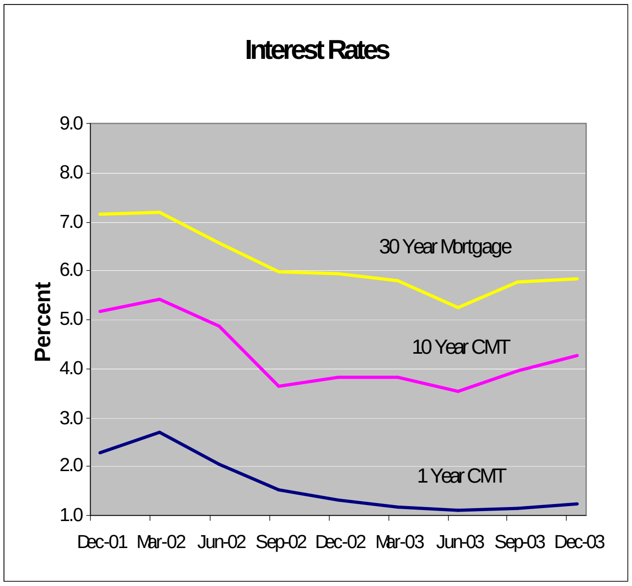
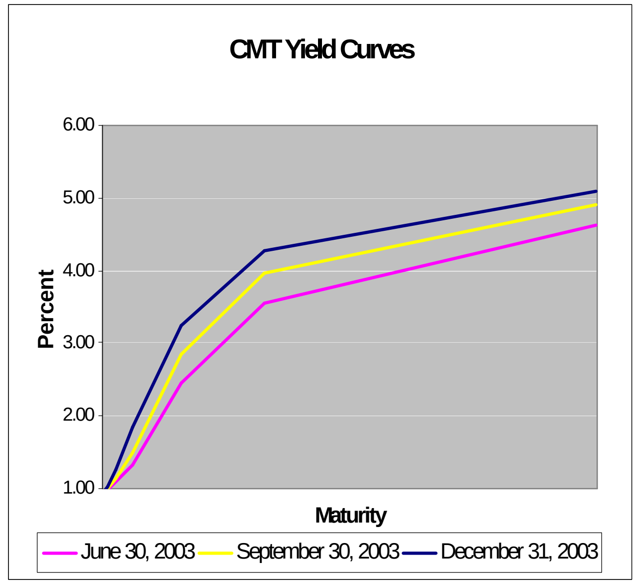
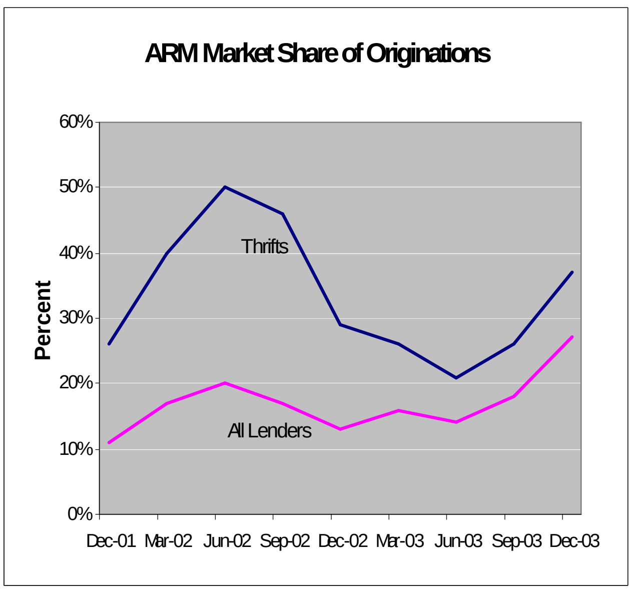
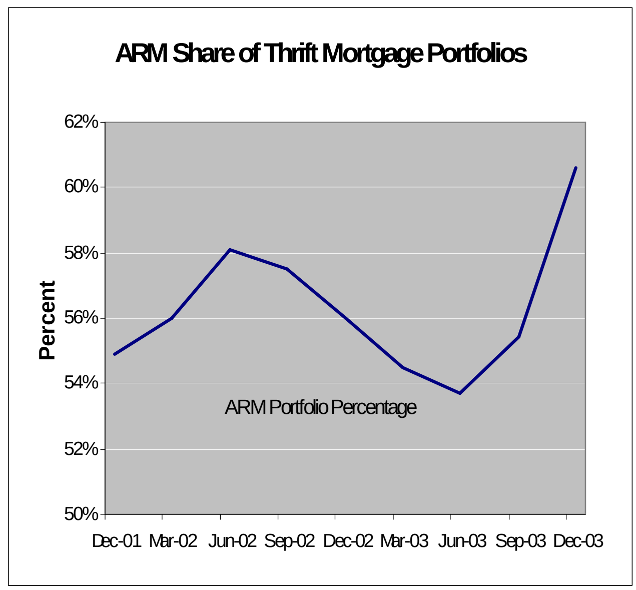
Interest Rates and ARM Market Share









Fourth Quarter Sees Interest Rate Sensitivity Rise (continued)

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lion, down from the record level of $230 billion in the third quarter.

This drop was due to the increase in interest rates and the consequent fall in the volume of mortgage refinancing activity in the fourth quarter.

Total mortgage originations in the fourth quarter were $163.9 billion, down sharply from $250.5 billion in the third quarter.

Thrifts' share of all 1-4 family originations was 23.4 percent in the fourth quarter, up from 19.2 percent in the third quarter. The rate of U.S. home ownership increased slightly to 68.6 percent, up from 68.4 percent in the third quarter. Refinancing accounted for 25.9 percent of thrift originations of single-family mortgages in the fourth quarter, down from 42 percent in the third quarter.

This substantial decrease is consistent with the refinancing activity of all lenders, where the rate fell from 68 percent to 49 percent between the third and fourth quarters.

The industry's average effective duration of assets rose from 1.81 to 1.92 between the third and fourth quarters. With the increase in interest rates in the fourth quarter, the NPV model predicted a decrease in the rate of prepayments of mortgages held in portfolio. This raised the average duration of mortgages

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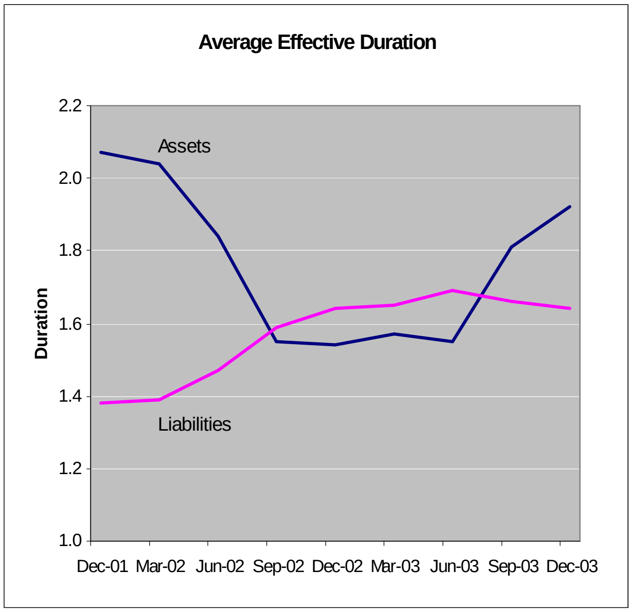
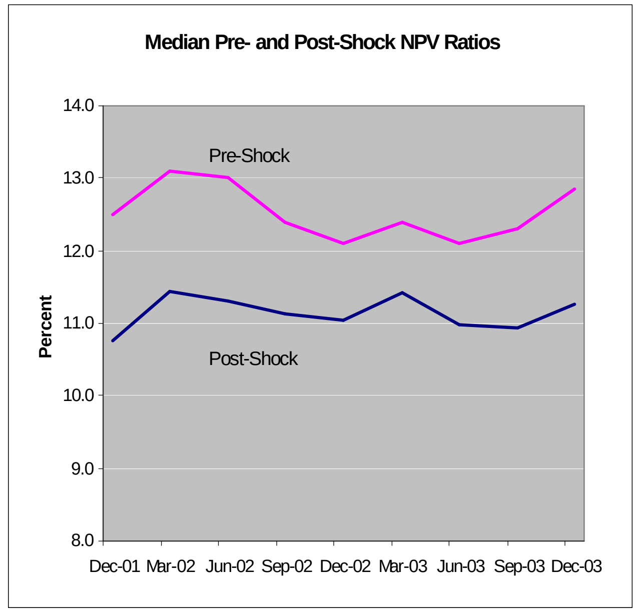
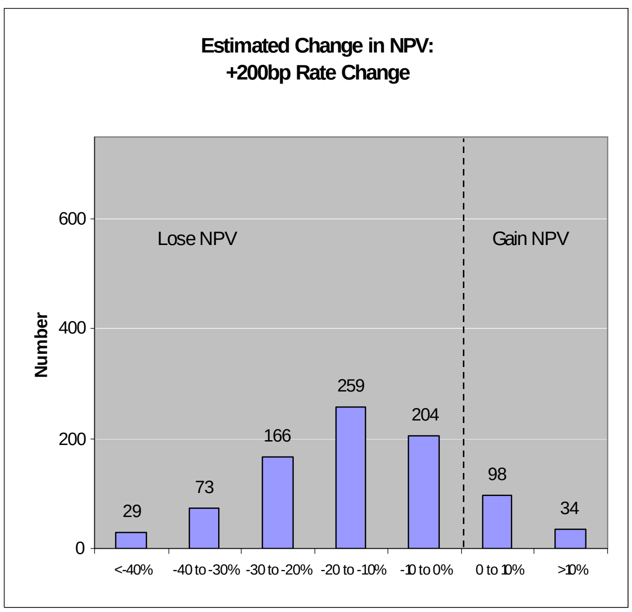
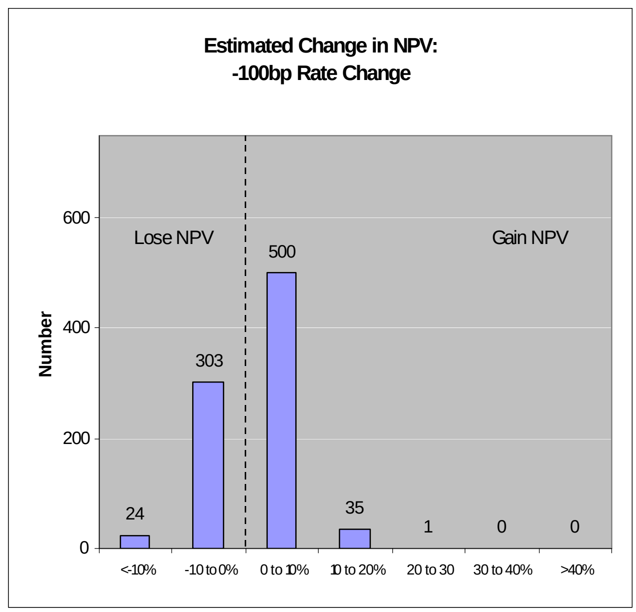
Duration and NPV Sensitivity Measures









Fourth Quarter Sees Interest Rate Sensitivity Rise (continued)

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and, therefore, total assets duration. The industry's average duration of liabilities fell slightly from 1.66 to 1.64 in the fourth quarter.

The changes in asset and liability durations in the fourth quarter produced an increase in the positive duration gap for the thrift industry as a whole. This is the second consecutive quarter that asset duration has exceeded liability duration, and the difference became larger.

While asset duration rose substantially in the fourth quarter, it is unclear that this upward trend will continue for another quarter.

This ambiguity is due to two countervailing factors. The first factor is that mortgage refinancings currently held in portfolio that were in the pipeline at the end of the fourth quarter will have a much lower likelihood of prepaying, since they will have much lower coupons than the mortgages they replaced. If interest rates remained unchanged, this would lengthen asset duration.

However, the second factor is that interest rates have already fallen dramatically so far in the first quarter of 2004. This would lead to higher prepayments and lower asset duration. As such, the net effect on asset duration is difficult to predict.

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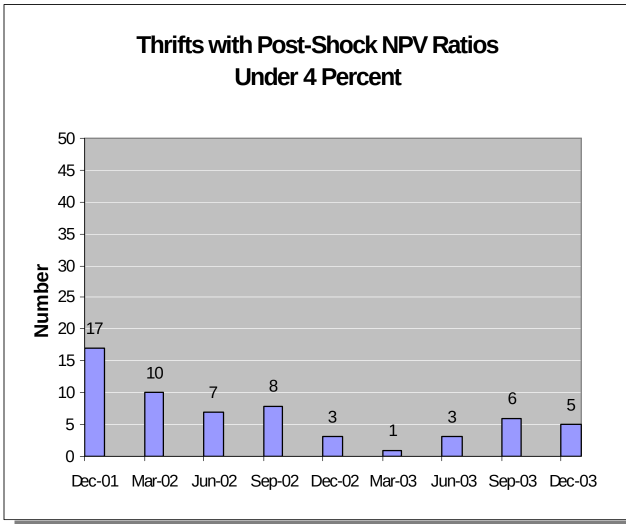
Interest Rate Risk Measures



Interest Rate Risk Measures
Industry Aggregates
Last Two Quarters

	NPV as % of PV of Assets		% Change in NPV	% Change in NPV
	Sep-03	Dec-03	Sep-03	Dec-03
+300	8.17%	8.09%	-24%	-29%
+200	9.06%	9.24%	-14%	-17%
+100	9.80%	10.20%	-6%	-7%
Base	10.33%	11.07%	0%	0%
-100	10.24%	10.87%	2%	3%
-200	N/A	N/A	N/A	N/A
-300	N/A	N/A	N/A	N/A

Post-Shock NPV Ratio and
Sensitivity Measure Matrix
September 2003

	Under 100bp	101-200bp	201-400bp	Over 400bp	Total
Over 10%	259	116	136	19	530
6% to 10%	112	87	88	12	299
4% to 6%	6	10	21	3	40
Below 4%	1	1	4	0	6
Total	378	214	249	34	875

Minimal | Moderate | Significant | High

Post-Shock NPV Ratio and
Sensitivity Measure Matrix
December 2003

	Under 100bp	101-200bp	201-400bp	Over 400bp	Total
Over 10%	227	148	158	26	559
6% to 10%	79	73	101	15	268
4% to 6%	3	7	18	3	31
Below 4%	0	1	4	0	5
Total	309	229	281	44	863

Minimal | Moderate | Significant | High

Fourth Quarter Sees Interest Rate Sensitivity Rise (continued)

(Continued from page 6)

The median pre-shock NPV ratio for the industry rose during the fourth quarter from 12.4 percent to 12.9 percent.

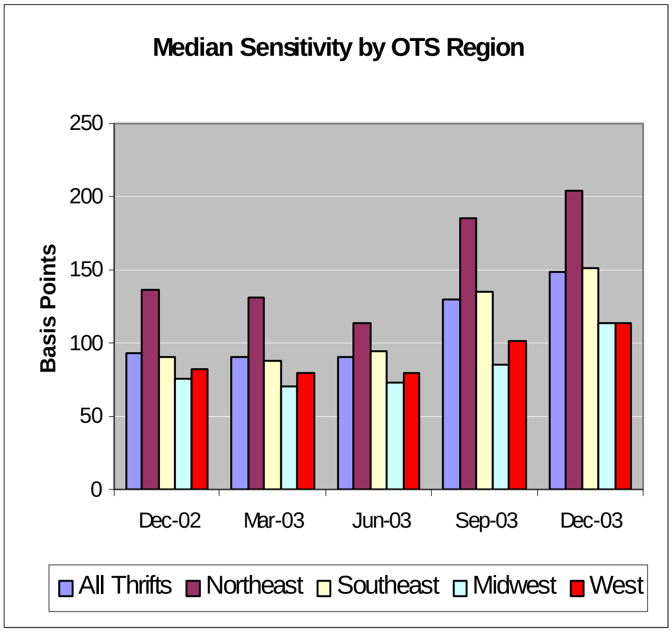
Along with this rise in the median pre-shock NPV ratio, the median post-shock NPV ratio also rose, moving from 11 percent at the end of the third quarter to 11.3 percent at the end of the fourth quarter.

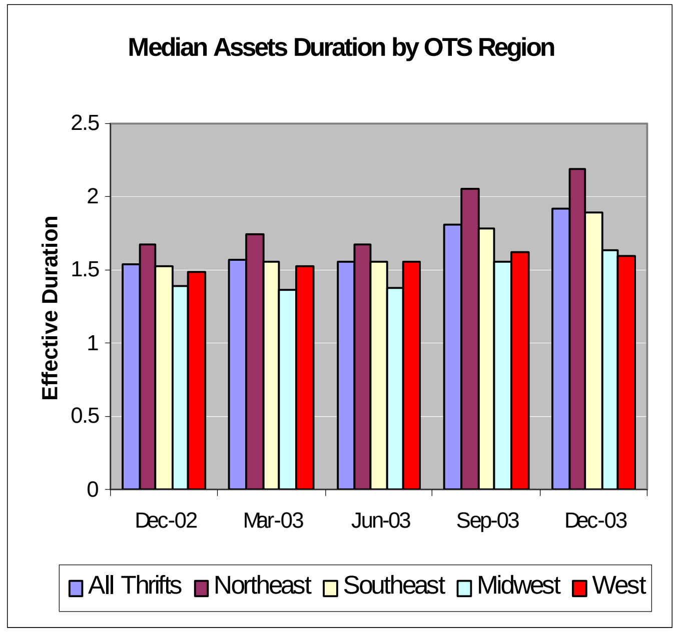
The number of thrifts with a post-shock NPV ratio below 4 percent fell to five institutions from six in the previous quarter.

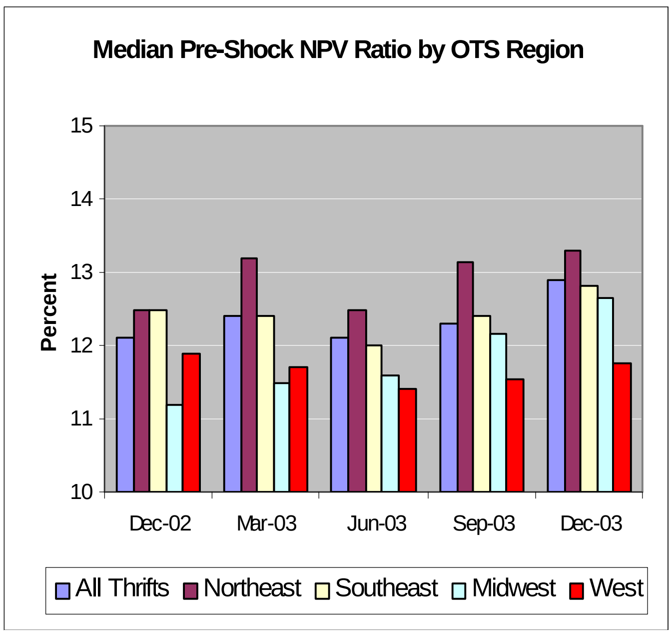
The percentage of thrifts with a post-shock NPV ratio over 6 percent increased between the third and fourth quarters. In the fourth quarter, such thrifts comprised 95.8 percent of the industry, compared to 94.7 percent in the prior quarter.

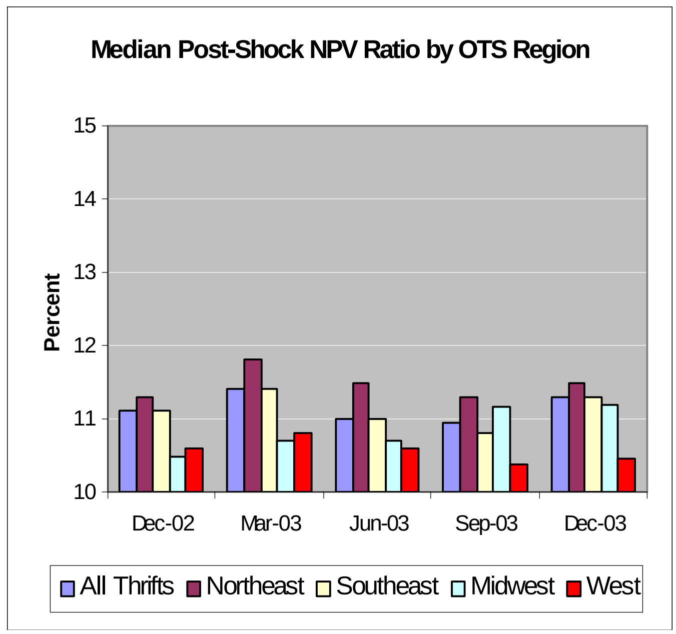
The number of thrifts with a post-shock NPV ratio below 6 percent fell to 36 in the fourth quarter, down from 46 in the third quarter. The percentage of thrifts with a sensitivity of 200 basis points or less decreased in the fourth quarter, falling to 62.3 percent from 67.6 percent in the prior quarter. In addition, the percentage of thrifts with over 400 basis points in sensitivity rose to 5.1 percent from 3.9 percent in the prior quarter. These results are consistent with the rise in median sensitivity for the industry in the fourth quarter. ¦

Comparative Trends in the Four OTS Regions









Regional Comparisons

The Northeast Region had the highest median sensitivity, at 204 basis points at the end of the fourth quarter, while the Midwest and West Regions had the lowest median sensitivity, at 113 basis points.

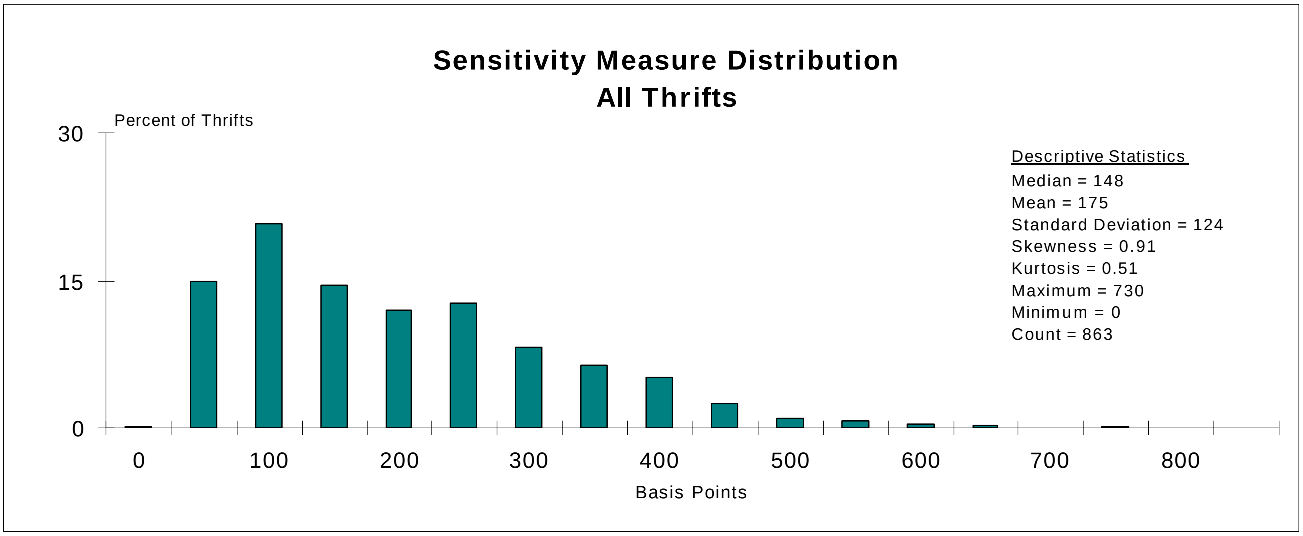
All OTS regions experienced increases in their interest rate sensitivity in the fourth quarter. The Midwest Region saw its median sensitivity rise by 32.9 percent, the largest relative increase of the four regions. The Northeast, Southeast, and West Regions saw their median sensitivities rise by 10.3 percent, 11.9 percent, and 10.8 percent, respectively.

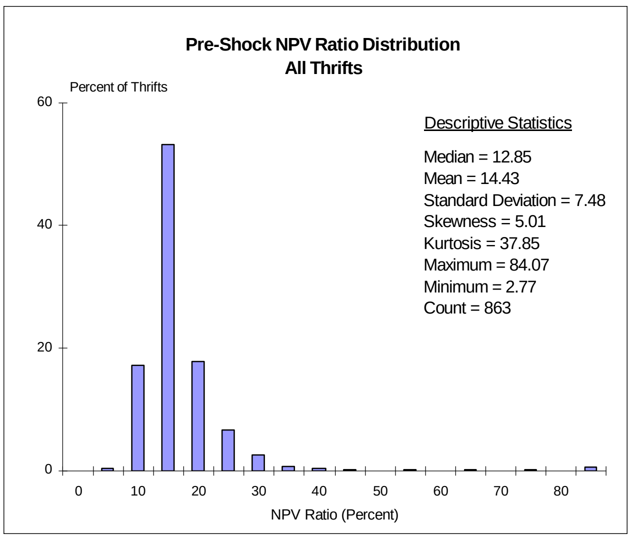
The Northeast Region had the highest median asset duration, at 2.19 at the end of the fourth quarter. The Northeast, Southeast, and Midwest Regions saw their median asset duration increase, while the West Region saw its median asset duration fall slightly from 1.62 to 1.60.

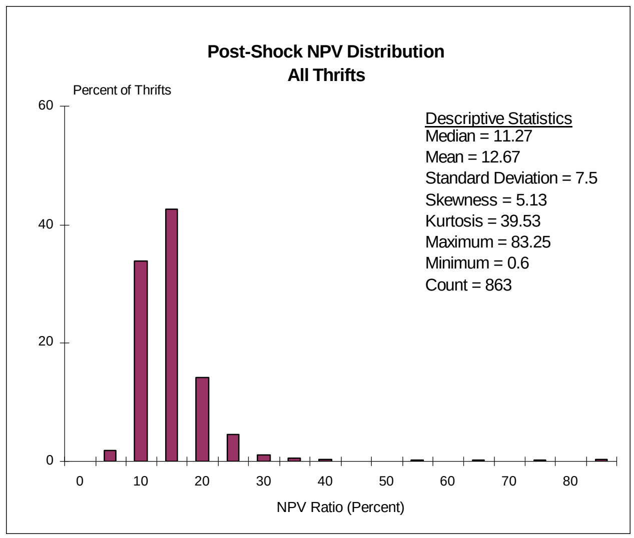
All OTS regions saw their median pre-shock NPV ratios rise in the fourth quarter. The Northeast Region had the highest pre-shock NPV ratio at 13.3 percent, while the West Region had the lowest pre-shock NPV ratio at 11.8 percent.

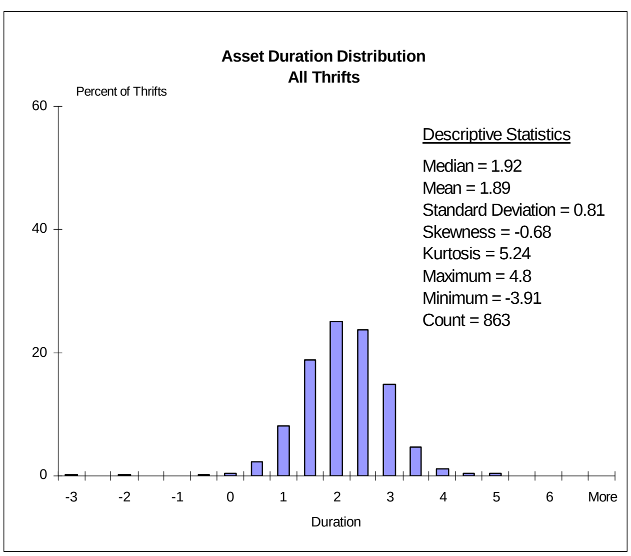
Median post-shock NPV ratios also rose in each of the four OTS regions in the fourth quarter.¦

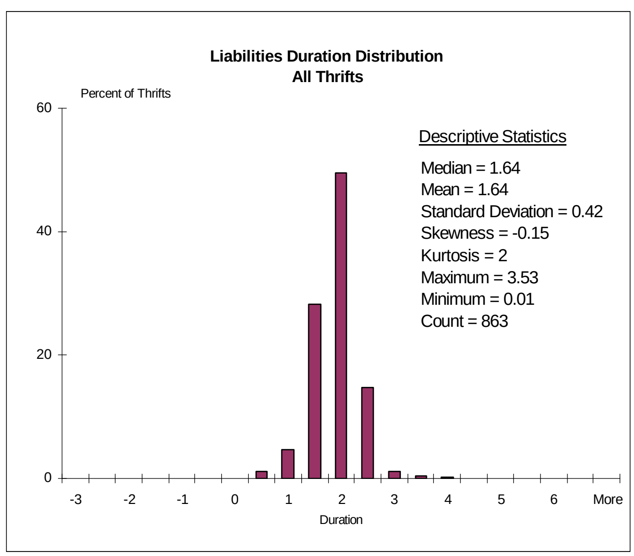
Appendix A — All Thrifts



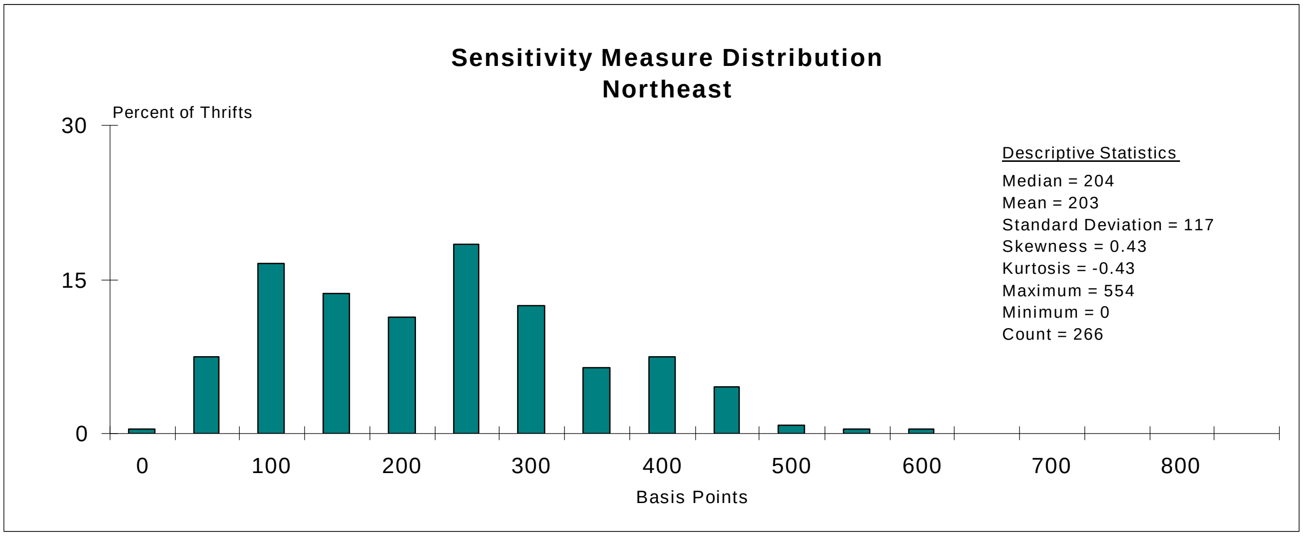


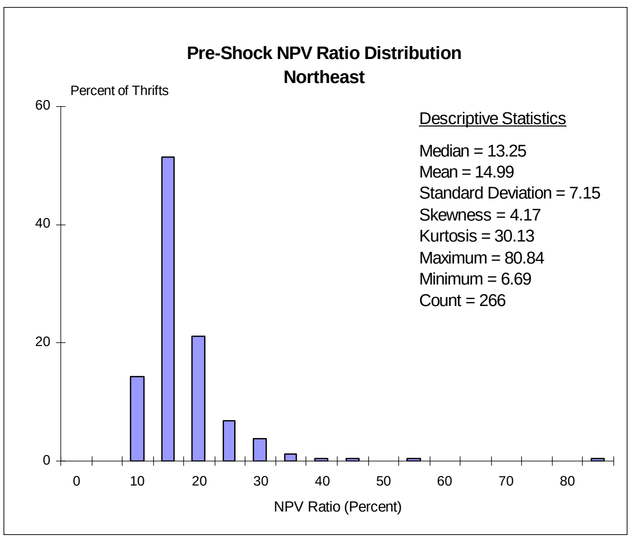


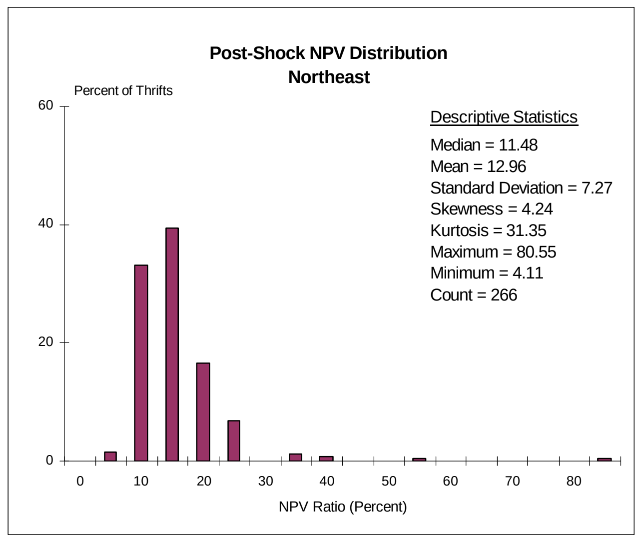


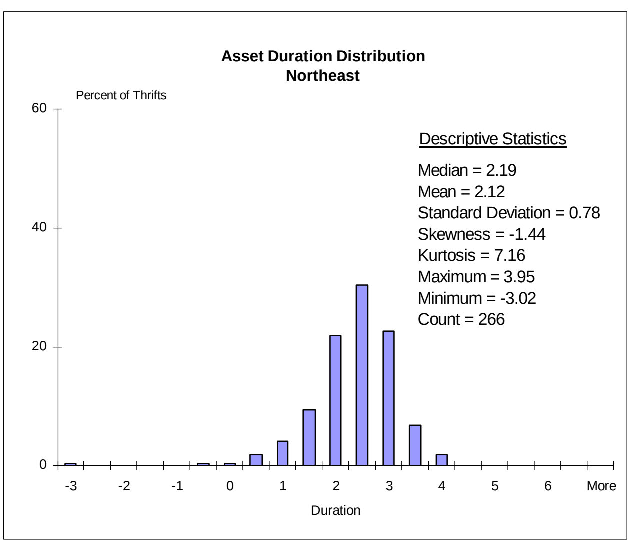


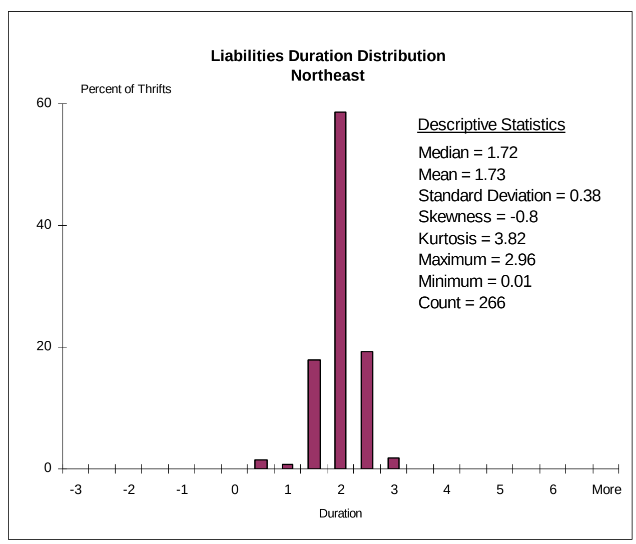
Appendix B — Northeast Region



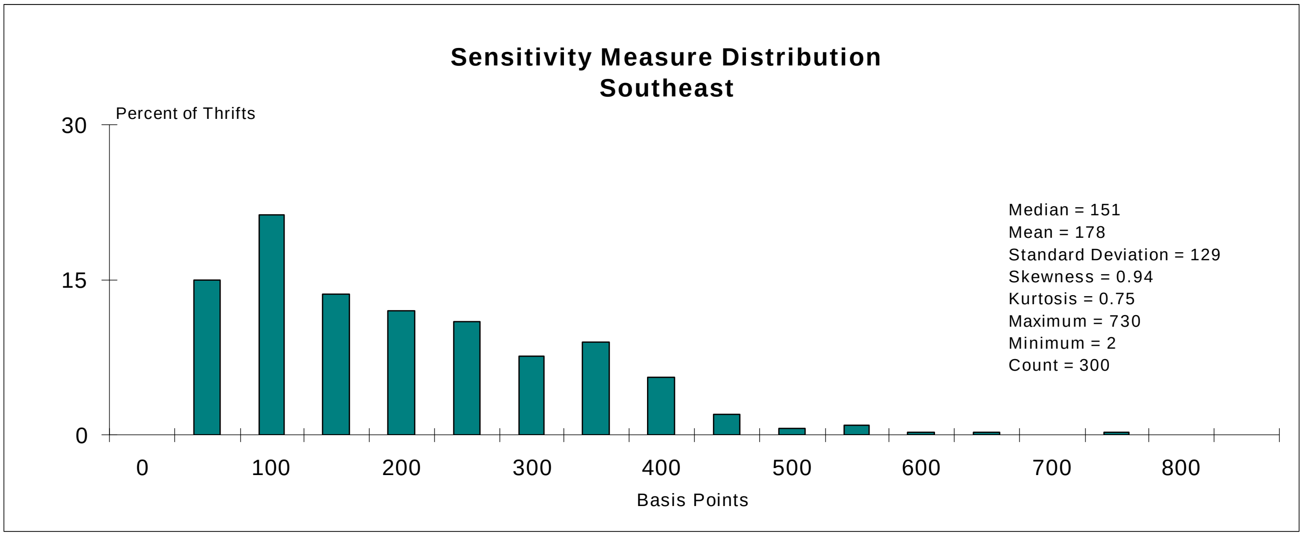


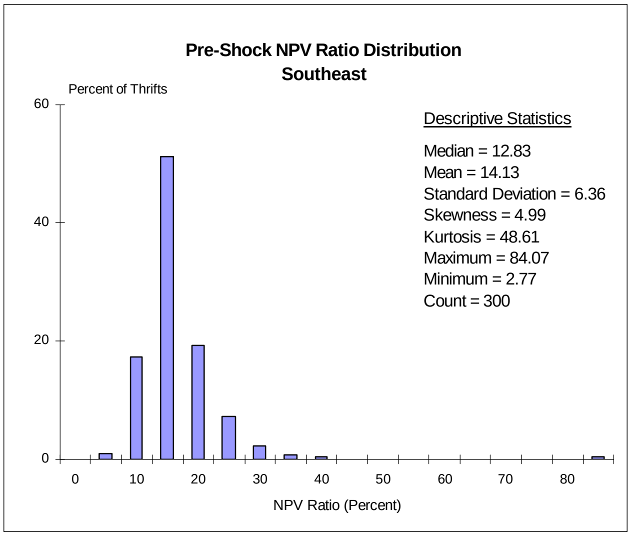


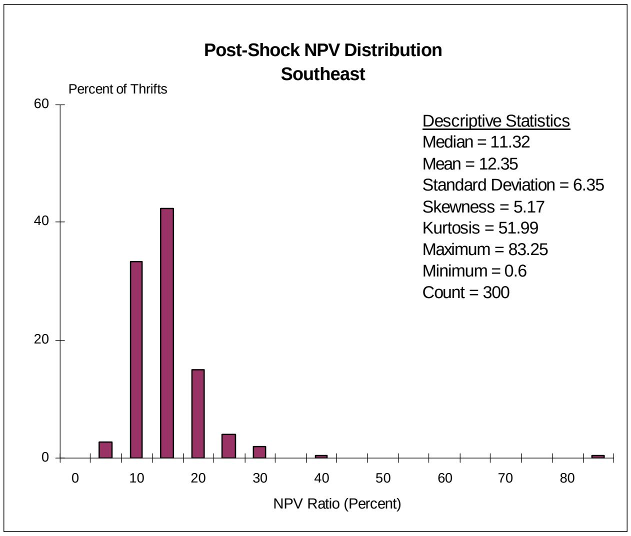


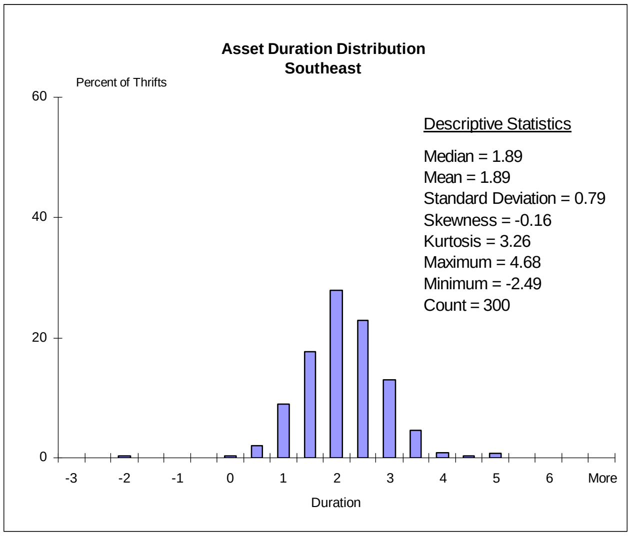


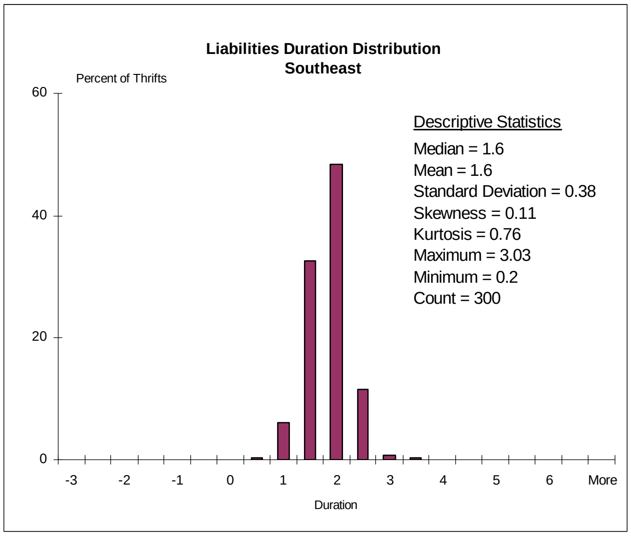
Appendix C — Southeast Region



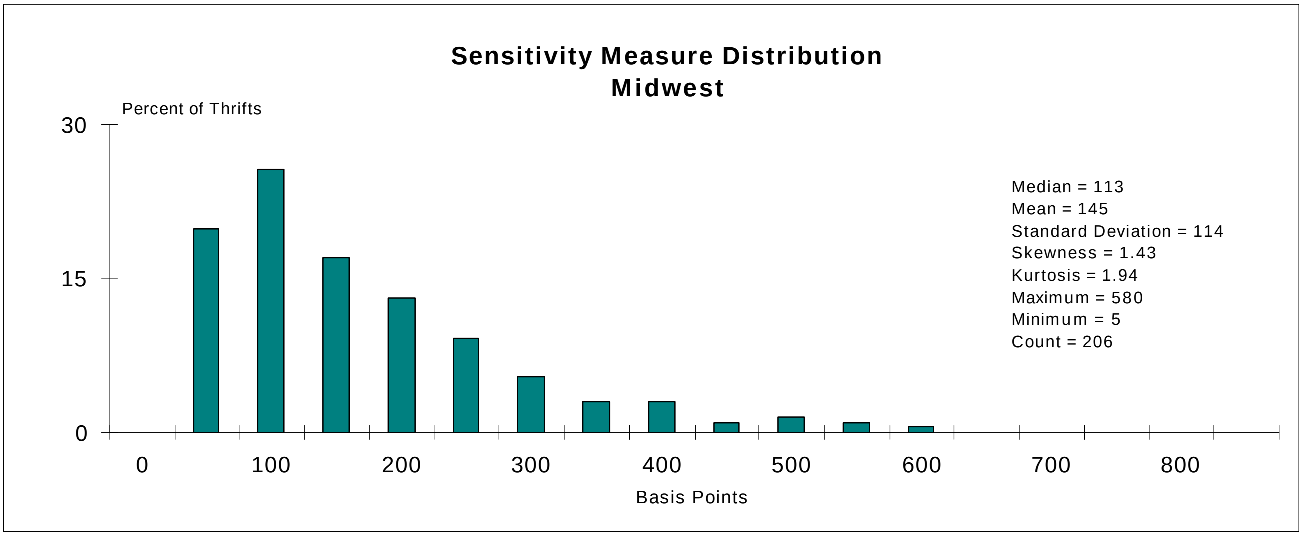


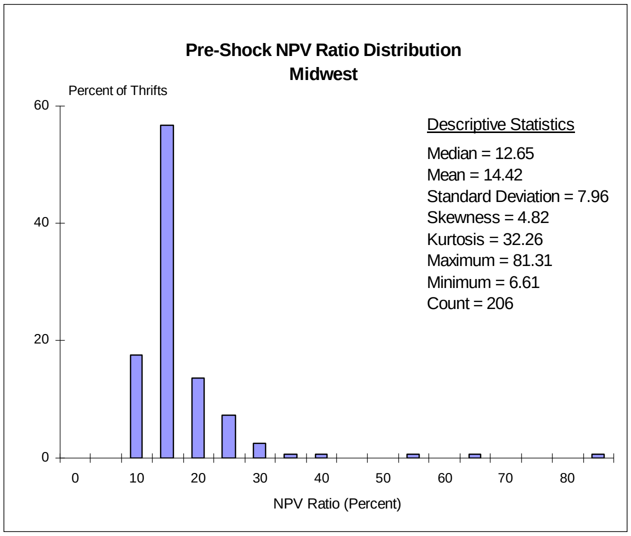


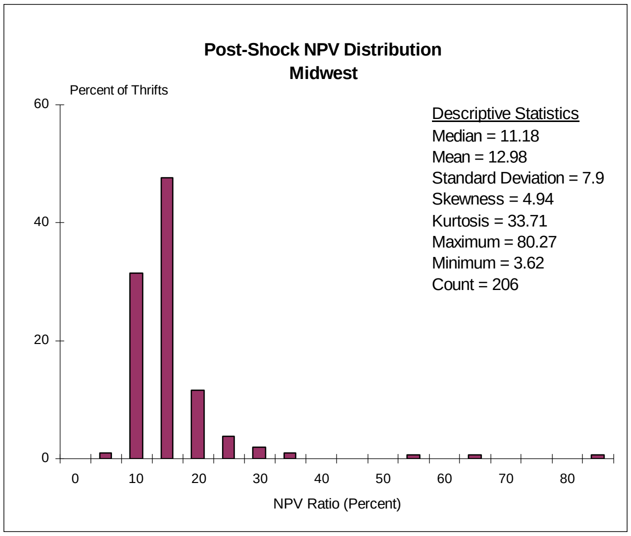


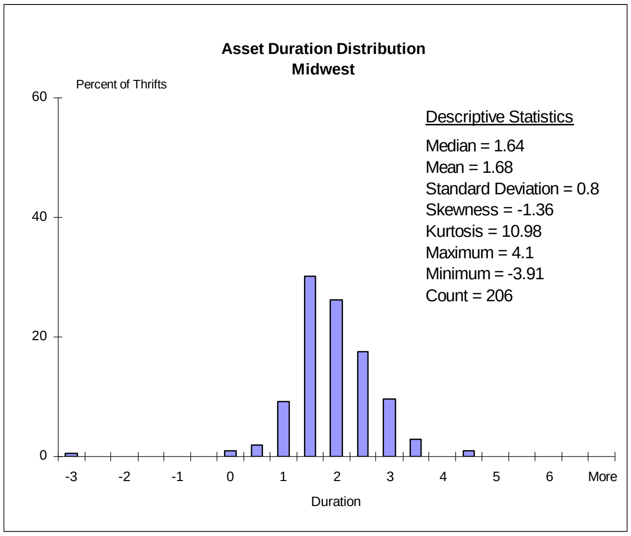


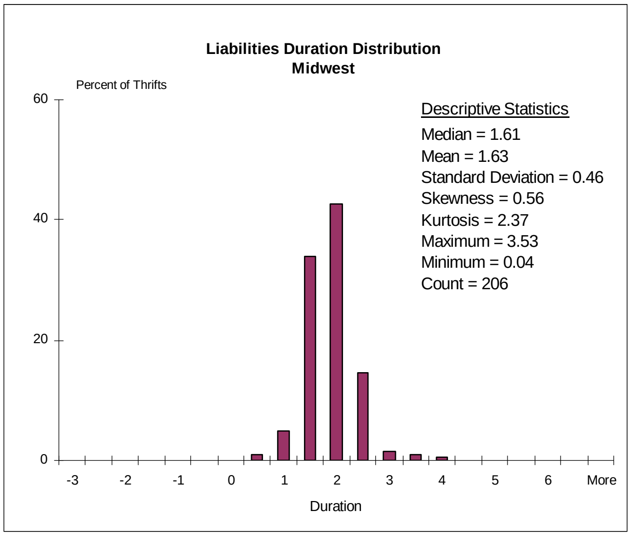
Appendix D — Midwest Region



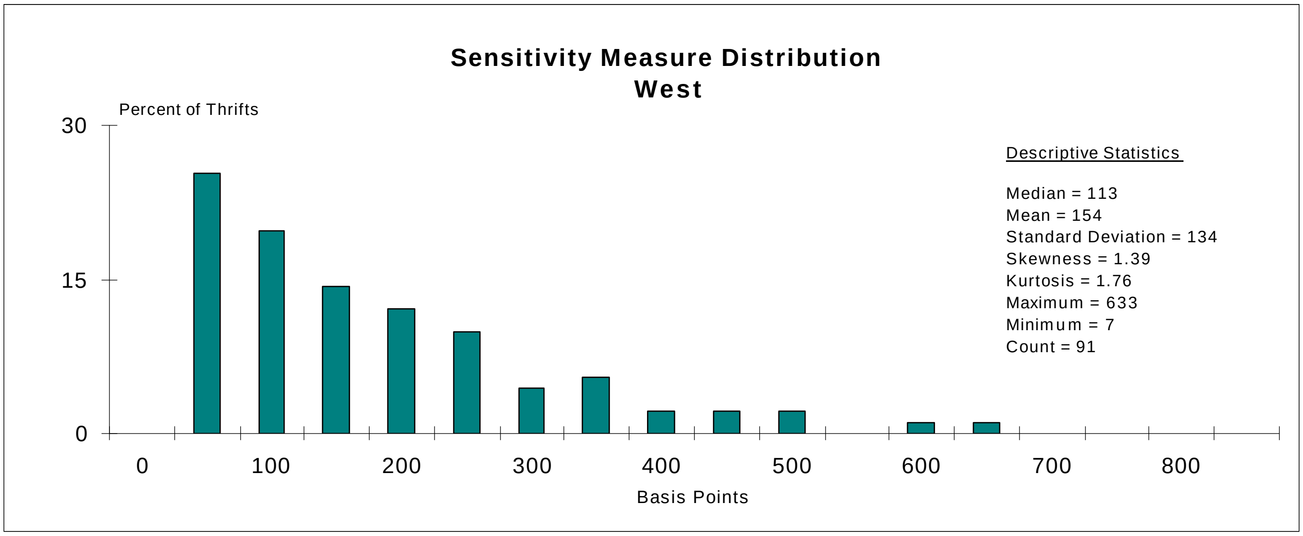


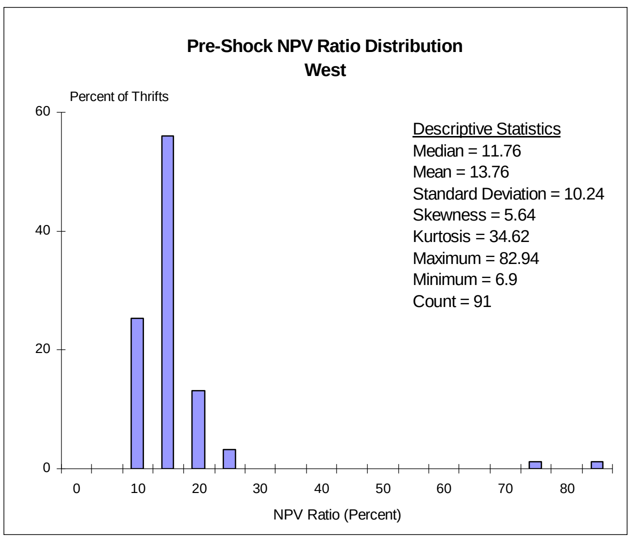


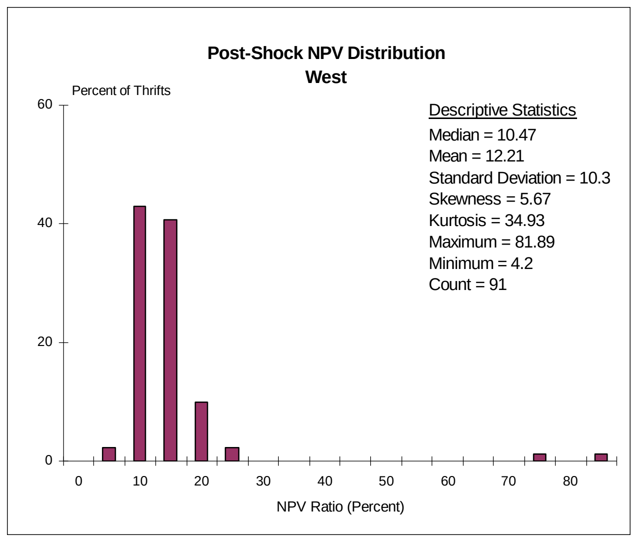


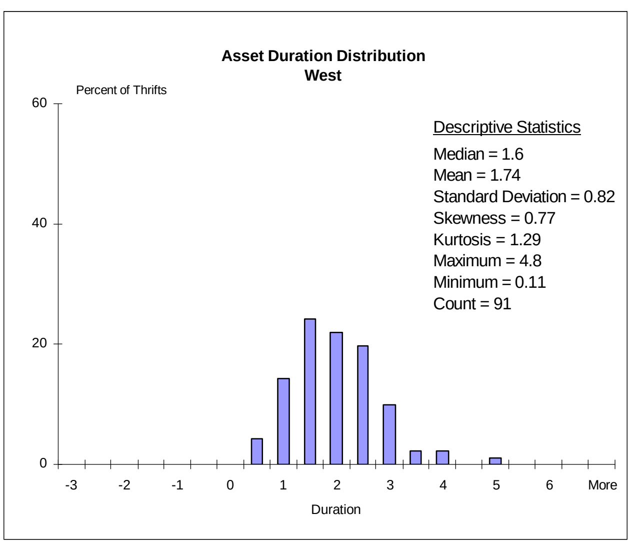


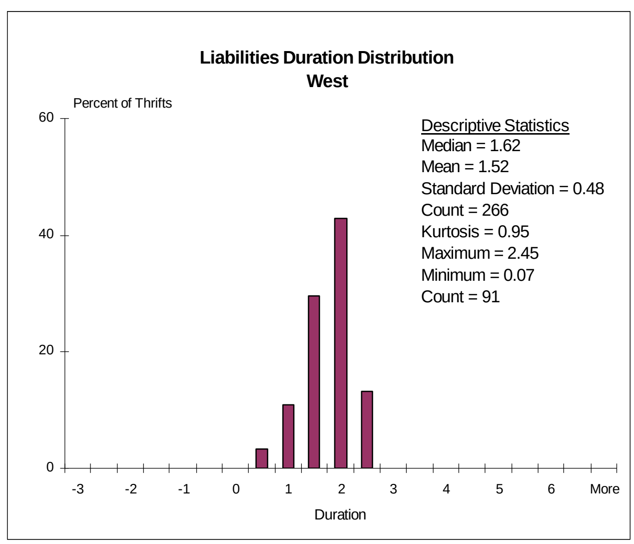
Appendix E — West Region











Glossary

Duration: A first-order approximation of the price sensitivity of a financial instrument to changes in yield. The higher the duration, the greater the instrument's price sensitivity. For example, an asset with a duration of 1.6 would be predicted to appreciate in value by about 1.6 percent for a 1 percent decline in yield.

Effective Duration: The average rate of price change in a financial instrument over a given discrete range from the current market interest rate (usually, +/-100 basis points).

Estimated Change in NPV: The percentage change in base case NPV caused by an interest rate shock.

Kurtosis: A statistical measure of the tendency of data to be distributed toward the tails, or ends, of the distribution. A normal distribution has a kurtosis statistic of three.

NPV Model: Measures how six hypothetical changes in interest rates (three successive 100 basis point increases and three successive 100 basis point decreases, assuming a normal interest rate environment) affect the estimated market value of a thrift's net worth.

Post-Shock NPV Ratio: Equity-to-assets ratio, following an adverse 200 basis point interest rate shock (assuming a normal interest rate environment), expressed in present value terms (i.e., post-shock NPV divided by post-shock present value of assets). Also referred to as the exposure ratio.

Pre-Shock NPV Ratio: Equity-to-assets expressed in present value terms (i.e., base case NPV divided by base case present value of assets).

Sensitivity Measure: The difference between Pre-shock and Post– shock NPV Ratios (expressed in basis points).

Skewness: A statistical measure of the degree to which a distribution is more spread out on one side than the other. A distribution that is symmetric will have a skewness statistic of zero.

Economic Analysis Division

Office of Thrift Supervision
1700 G Street, NW
Washington, DC 20552

David Malmquist, Director
Economic Analysis Division
Phone: 202-906-5639
Email: david.malmquist@ots.treas.gov

Prepared by:

Jonathan D. Jones
Economic Analysis Division
Phone: 202-906-5729
Email: jonathan.jones@ots.treas.gov

Robert Sutter, IT Specialist, assembled the data reported in the Appendices.

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